Eigenvalue Statistics of Reduced Density Matrix during Driving and Relaxation

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(Received 14 February 2013; published 14 May 2013)

We study a subsystem of an isolated one-dimensional correlated metal when it is driven by a steady electric field or when it relaxes after driving. We obtain numerically exact reduced density matrix \( \rho \) for subsystems which are sufficiently large to give significant eigenvalue statistics and spectra of \( \log(\rho) \). We show that both for generic as well as for the integrable model, the statistics follows the universality of Gaussian unitary and orthogonal ensembles for driven and equilibrium systems, respectively. Moreover, the spectra of modestly driven subsystems are well described by the Gibbs thermal distribution with the entropy determined by the time-dependent energy only.

DOI: 10.1103/PhysRevLett.110.200602 PACS numbers: 05.70.Ln, 05.30.-d, 71.27.+a, 72.10.Bg

Introduction.—Spectral universality is one of the key features of highly excited complex systems. It has been demonstrated and observed in a diverse range of phenomenologies ranging from acoustics [1], microwave resonators [2], and quantum dots [3], to many-particle systems such as complex nuclei [4] and strongly correlated models of condensed matter [5]. Universality is quantitatively characterized by the applicability of a parameter free random matrix theory (RMT) [6], where the Hermitian operator in question, usually the Hamiltonian, is described by an ensemble of Gaussian random matrices, where the only constraint, whether matrices are real symmetric or complex Hermitian, is imposed by the existence or nonexistence of a (generalized) time-reversal symmetry. Random matrix distribution of energy levels is also widely used as a clean indicator of complexity or nonintegrability of a physical model and is the most abstract definition of a quantum chaotic behavior [7].

In this Letter, we propose RMT analysis of a completely different concept in quantum statistical physics, namely, of spectra of reduced density matrix (RDM) \( \rho \) of equilibrium and nonequilibrium states. We consider RDM of strongly correlated quantum systems. In particular, we study the one-dimensional (1D) model of interacting spinless fermions (equivalent to a Heisenberg-type spin chain) for a variety of simple pure states of the entire system: the so-called microcanonical (MC) states (approximate eigenstates), the time-evolving states after a quench of magnetic flux, or during inductive driving with a linearly increasing magnetic flux. We show that quite remarkably the statistics of eigenvalues of RDM of large subsystems is typically described by RMT. For equilibrium thermal states, we find agreement with Gaussian orthogonal ensemble (GOE), whereas for nonequilibrium driven states with currents, we find agreement with Gaussian unitary ensemble (GUE). We note in particular that spectra of RDM of large subsystems typically follow RMT even if the entire system is completely integrable.

Furthermore, the RDM can serve as a stringent test of thermal properties of nonequilibrium states and their thermalization. Our results show that for modestly driven systems the entropy density \( s \) of the subsystem develops in time according to the quasiequilibrium scenario; i.e., \( s \) depends only on the instantaneous energy density \( \varepsilon \). Moreover, we demonstrate for driven integrable and nonintegrable chains that the eigenvalue spectra are consistent with the canonical Gibbsian form \( \rho \propto \exp(-H_{\text{eff}}/T) \) with well-defined effective temperature \( T \) and \( H_{\text{eff}} \) being a \( T \)-independent effective Hamiltonian of the subsystem.

Model and method.—We study the 1D model of interacting spinless fermions on a chain of an even number of sites \( L \) with periodic boundary conditions. We investigate how the system responds to an external electric field as introduced in the time-dependent model by the varying magnetic flux \( \phi(t) \),

\[
H(t) = -t_0 \sum_j \{ e^{\phi(t)/L} c_{j+1}^\dagger c_j + H.c. \} + V \sum_j \hat{n}_j \hat{n}_{j+1} + W \sum_j \hat{n}_j \hat{n}_{j+2}, \quad (1)
\]

where \( \hat{n}_j = c_j^\dagger c_j \), \( t_0 \) is the hopping integral, and \( V \) and \( W \) are the repulsive potentials between fermions on the nearest-neighbor and the next-nearest-neighbor sites. Further on, we use units in which \( \hbar = k_B = t_0 = 1 \). The main idea behind introducing \( W \) is to break integrability of the pure \( t_0 - V \) model. One expects generic properties for the nonintegrable case with \( W \neq 0 \), whereas the integrable system (\( W = 0 \)) shows anomalous relaxation [8–11] and transport characteristics [12–18]. If not stated otherwise, the numerical results for integrable and nonintegrable cases will refer to \( V = 1, W = 0 \), and \( V = 1.4, W = 1 \) systems, respectively, at half filling with \( M = L/2 \) fermions on \( L = 26 \) sites. These parameters correspond to the metallic regime.